Semi-Blind Power Allocation for Digital Subscriber Lines

Rodrigo Moraes*, Boris Dortschy†, Aldebaro Klautau* and Jaume Rius i Riu†
*Signal Processing Laboratory, Federal University of Pará, Belém – PA, Brazil
†Broadband Access Research (BAR), Ericsson EAB, Kista, Sweden
rbmoraes@cetuc.puc-rio.br, aldebaro@ufpa.br, {boris.dortschy, jaume.rius.i.riu}@ericsson.com

***Abstract*— Digital subscriber lines (DSL) are today one of the most important means for delivering high-speed data transmission. An emerging technique for dealing with one of the technology's most harmful problems, crosstalk, is dynamic spectrum management (DSM). DSM literature already counts with some half a dozen important solutions. These solutions can be classified according to four different aspects: optimality, computational cost, distribution and required crosstalk channel information. In this work we present an algorithm, named semi-blind spectrum balancing, which achieves a compelling trade-off between these four aspects. The scheme is based on the idea of optimization against a *virtual line*, a fictitious line to represent the damage caused to other users in the network. This line is adjusted with the aid of limited message-passing between modems and a central agent and very simple crosstalk channel information. Crosstalk channel knowledge required should be much simpler to obtain than full channel estimation.**

I. INTRODUCTION

Digital subscriber lines have been a subject of great interest in both industry and academia in the past few years. The ubiquity of the telephone network is likely its most attractive feature in the eyes of both companies and users. However, DSL operates in a media which was not primarily designed to carry high speed digital data transmission, which gives rise to crosstalk. For DSL frequencies of up to several MHz, crosstalk really becomes an issue.

Strategies to decrease crosstalk influence which deal with spectra design have been named dynamic spectrum management (DSM). DSM literature already counts with about half a dozen important solutions. These different methods can be qualified in four aspects: rate performance (rate region), centralization, computational cost and required crosstalk channel knowledge. A method with good results in all four categories remains yet an open problem. This work tries to cope with this need. The method to be described, called Semi-Blind Spectrum Balancing (2SB), achieves good balance among these four characteristics. 2SB uses the idea of a *virtual line* (VL) to represent the damage to be caused to other lines. The VL should be adapted with the aid of limited information exchange between modems and a central agent and very simple crosstalk channel knowledge.

The remaining of this work is outlined as follows. Section II presents the problem of interest and previous solutions. The method is introduced in Section III, followed by some simulation results in Section IV. Final remarks are made in Section V.

II. PROBLEM STATEMENT AND PREVIOUS SOLUTIONS

Most types of DSL standards, such as ADSL and VDSL, adopted discrete multitone (DMT) [1], which is the modulation technique assumed in this work. The idea that constitutes the core of this technique is the division of the available spectra in a number of Δ_f-spaced independent sub-channels or tones. For a matter of simplicity, it is considered that DMT tones do not experience inter-carrier interference (see, e.g., [2]).

Given a N-user DMT system with K tones, let $\mathbf{P}_{(N\times K)} = \{p_n^k\}$ be a matrix in which p_n^k is the transmitter PSD of the n-th user on tone k. The n-th row $\mathbf{p}_n = [p_n^1, \cdots, p_n^K]$ represents the power allocation of user n across all tones and the k-th column $\mathbf{p}^k = [p_1^k, \cdots, p_N^k]$ contains the PSD levels of all users in a given tone k. Let σ_n^k be the background noise's PSD of the n-th user on tone k. Also, let h_{mn}^k be the crosstalk transfer function between transmitter m and receiver n at tone k. Both σ_n^k and h_{mn}^k are normalized by being divided by the direct channel of user n, $|h_{nn}^k|^2$. The bit loading for user n on tone k is defined as

$$b_n^k = \log_2\left[1 + \frac{1}{\Gamma}\frac{p_n^k}{\left(\sigma_n^k + \sum_{m\neq n} p_m^k |h_{mn}^k|^2\right)}\right], \qquad (1)$$

where Γ is the gap, which depends on the desired BER, coding gain and noise margin [1]. Each b_n^k is a function of its respective column vector $\mathbf{p}^k$. The total bit rate of line n is $R_n = f_s \sum_k b_n^k$, where f_s is the symbol rate. Throughout this work the practical issue of loading a tone with a integer number of bits is ignored.

The problem of interest is that of finding a $\hat{\mathbf{P}}$ matrix which maximizes data rates of all users in the network given a power budget for each user. This problem has been often referred to in literature as Rate Adaptive (RA) problem [3]. In mathematical form it can be written as the maximization of the rate of one user in the network (in our notation, this will always be user 1) while the others achieve a minimum specified data rate,

$$\hat{\mathbf{P}} = \arg\max_{\mathbf{P}} R_1 \qquad (2)$$

$$\text{s.t.} \quad P_n^{\text{tot}} \le P_n^{\text{max}}\ \forall n \quad \text{and} \quad R_n \ge R_n^{\text{min}}\ \forall n > 1\ ,$$

in which P_n^{tot} and P_n^{max} are the total actually allocated power and maximum power available to user n, respectively; and R_n and R_n^{min} are the achieved and minimum desired rate for user n, respectively.

The optimization problem in (2) can be rewritten as a weighted rate sum maximization [4],

$$\hat{\mathbf{P}} = \arg\max_{\mathbf{P}} \sum_n w_n R_n \tag{3}$$

$$\text{s.t.} \quad P_n^{\text{tot}} \leq P_n^{\text{max}} \ \ \forall n \quad \text{and} \quad R_n \geq R_n^{\text{min}} \ \forall n > 1 \ ,$$

in which the w_n values can be interpreted as a priority given to each user. They should be adjusted such that priorities are just enough for achieving minimum rates for all $n > 1$ users. For convenience, it is considered that $\sum_n w_n = C$, where C is a constant. If $C = 1$, the w_n's are more straightforwardly interpreted as proportions. The first user, which should have its rate maximized, should get the "rest", $w_1 = 1 - \sum_{n>1} w_n$.

The optimization in (2) is non-convex in $\mathbf{P}$, containing many local extrema. This makes its solution non-trivial. Good results are not found by simple search procedures. The rate and power constraints also couple power allocation across tones and users, complicating the problem even further.

One of the first successful solutions to the DSM problem was IWF [5]. An unquestionable improvement came with OSB [4]. OSB presented a great increase in rate performance, but also great demands on the other three aspects. Lately focus has been on how to cope with OSB's limitations and profit from the insight it proportioned to come with more viable schemes.

Three solutions deserve attention in this context. The earliest attempt was ISB [4], [6], followed by SCALE [7], and ASB [8].

The kind of static pricing adopted in ASB is important to this work. The innovation in ASB was to introduce the concept of a reference line. The reference line of user n is a fictitious line that works as a type of pricing representing the damage to be caused to all other users. A local optimization of each user against its respective reference line is claimed to provide a good equilibrium between local and global optimization. The reference line parameters are static and are considered to be an a-priori information to be loaded in modems before transmission. There are two problems with this: first, the static reference line does not take into account the dynamic nature of DSL channels. Although channels gains vary very slowly with time, network behavior is constantly changing; and, second, coming up with such a-priori information demands reliable crosstalk channel information. That can be achieved either through channel measurements, which are complicated, or heuristic calculation, which are known to highly inaccurate.

As said, this work will classify DSM schemes in four different aspects. Table I summarizes such classification. The challenge is to come up with a good tradeoff between the four of them.

III. Proposed Solution: 2SB

2SB builds up in knowledge about the present network state with a series of message-passing between modems and a central agent with limited amount of information. Modems make an optimization against the already introduced VL. The VL has a similar purpose to that of the reference line, but it is not static. Through the development of iterations a spectrum management center (SMC) is responsible for suggesting new VL parameters to each modem according to the present network state. In this way, the dynamic nature of DSL channels is considered and the network adjusts itself independently of a-priori information to a more profitable state, in which each modem is both aware of its rate requirements and its impact on other lines. 2SB is self-adjusting enough as to avoid explicit per-tone crosstalk channel knowledge. Information about crosstalk channels has to exist, but, as will be seen, it can be rough enough to avoid channel measurement campaigns or usage of inappropriate methods for crosstalk channel estimation.

TABLE I

Comparison of previous solutions

	Rate Performance	Computational Cost	Centralization	Xtalk Chan. Info
IWF	Poor	Low	None	None
OSB	Optimal	High	Full	Yes
ISB	Optimal	Medium	Full	Yes
SCALE	Good	Low	Semi	Yes
ASB	Good	Low	None	Yes

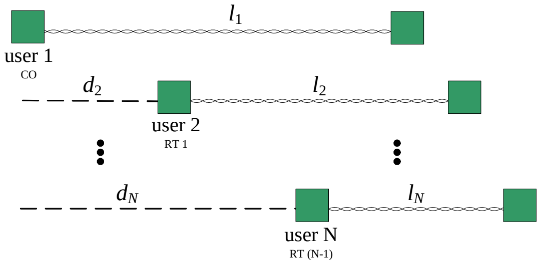

Fig. 1. Scenario: $l_1 \geq l_2 \geq \ldots > l_N$ and $d_N \geq \ldots \geq d_2$.

To illustrate how the algorithm works, the N-user downstream ADSL scenario depicted in Fig. 1 will be considered. In this scenario, the lengths of the lines obey the relation $l_1 \geq l_2 \geq \ldots \geq l_N$ and the distance from remote terminals (RTs) to central office (CO) obey $d_N \geq \ldots \geq d_2$. This kind of topology is one of the most challenging for spectrum management, since the receiver of the n-th user experiences a large amount of crosstalk originated from the transmitters of users $n+1$ to N, RT n to RT $(N-1)$, respectively. All users have minimum rates except user 1, which is to have its rate maximized.

The complete method is described in Alg. 1. It consists mainly in four steps: power allocation in modems; modems sending crosstalk information to the SMC; processing in the SMC; and SMC sending updated VL parameters to modems. These four steps should be repeated until convergence. Each of these will be detailed in a separate section.

A. Power Allocation

The first step is to define the VL mathematically. As said, this line should represent the damage to be caused to all other users. All users should have an independent VL, with different

Algorithm 1: 2SB

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Input: $P_n^{\max}\ \forall n; R_n^{\min}\ \forall n > 1, ; \sigma_n^k, h_{nn}^k, \tilde{\sigma}_n^k\ \forall n, k$
Output: $\hat{\mathbf{P}}$
1:  Set $\tilde{p}_n^k$ and $\tilde{h}_n^k$ to flat levels $\forall n$;
2:  Repeat
3:    For $n = 1, \ldots N$,
4:      Modems: Optimization against VL (Eq. 5);
5:    Modems: Send $\text{CDR}_n^k\ \forall n, k$ and $w_n\ \forall n$;
6:    SMC: Processing
7:    Choose $l^k$ as the second greater $w_n$ of all $n$ that allocate
      power on $k\ \forall k$;
8:    For $i = 1, ..., N$, ($i$ stands for interferer, $v$ for victim)
9:      For $k = 1, ..., K$,
10:       $\text{mCDR}_i^k = \max_{v \neq i}\ c_{iv} w_v \text{CDR}_v^k$ ;
11:       $f^k = e^{\frac{-(\text{mCDR}_i^k - l^k)^2}{2v^2}}$;
12:       If $\text{mCDR}_i^k > l^k$,
13:         $\tilde{h}_i^k = \tilde{h}_i^k (2 - f^k)$;
14:       Else
15:         $\tilde{h}_i^k = \tilde{h}_i^k / \alpha$;
16:   SMC: Send $\tilde{h}_n^k\ \forall n, k$;
17: until PSD convergence.
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channel conditions and achieved rate. The VL is defined by three parameters: PSD $\tilde{p}_n^k$, channel gain $\tilde{h}_n^k$ and background noise $\tilde{\sigma}_n^k$. Bit loading for the VL will be calculated by

$$\tilde{b}_n^k = \log_2 \left[1 + \frac{1}{\Gamma} \frac{\tilde{p}_n^k}{\tilde{\sigma}_n^k + \tilde{h}_n^k p_n^k} \right]. \tag{4}$$

Power allocation for every tone and user should be found with an optimization of each user against its respective VL (step 4 in Alg. 1). The optimization should maximize the data rate of the VL, or, in other words, minimize the damage to be caused to it. In this fashion, a good equilibrium between network and individual performance is achieved. The problem each modem should solve is

$$\hat{p}_n^k = \arg\max_{p_n^k} \left[w_n b_n^k + (1 - w_n) \tilde{b}_n^k - \lambda_n p_n^k \right], \tag{5}$$

in which w_n is the weight or priority user n has in respect to the VL. Users 2 to N should find the minimum possible w_n, which ranges from 0 to 1, so that their minimum data rates are achieved. User 1, the CO, is to have its rate maximized and thus $w_1 = 1$ (the VL does not play a role in the CO power allocation, and it basically does waterfilling). The λ_n are Lagrangian variables which serve as control for power. They should be adjusted such that $P_n^{\text{tot}} = P_n^{\max}$. It is considered that power allocation is done in parallel – all users allocate power at the same time.

The VL's PSD and channel gain are initialized with flat values. These are the values that will determine the power allocation in the first iteration. They do not faithfully represent the real crosstalk situation in the binder and hence the resulting $\hat{\mathbf{P}}$ matrix of the first iteration will be fairly poor in terms of Eq. (2). This situation should be adjusted so that the VLs for all users represent more accurately the actual noise environment in the binder. The VL's background noise of user n should be set as to resemble the background noise of the users it interferes upon. We now turn attention to message-passing.

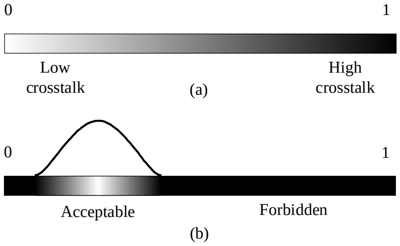

Fig. 2. Crosstalk Damage Ratio.

B. Message-Passing I

By the end of every power allocation, modems should report to the SMC the crosstalk damage they received (step 5 in Alg. 1). These damage will be assessed by the *crosstalk damage ratio* – CDR. The CDR is defined as

$$\text{CDR}_n^k = 1 - \frac{b_n^k(\text{xt} + \sigma)}{b_n^k(\sigma)}, \tag{6}$$

where $b_n^k(\text{xt} + \sigma)$ is the bit loading of user n on tone k when considering background noise plus crosstalk and $b_n^k(\sigma)$ is the bit loading when considering only background noise. It is considered that every user has a fixed estimative of its background noise.

The CDR ranges in a continuum from 0 to 1. The closer it is to zero, the less significant the crosstalk damage. The closer it is to 1, the higher it is the crosstalk damage. Consider a simple example: a given user on a specific tone allocates a certain amount of power which, when considering background noise only, amounts to 10 bits. If, when considering crosstalk, bit loading goes down to 8 bits, CDR would equal 0.2, which could mean that crosstalk in this tone is bearable. If bit loading drops to 2 bits, CDR would be 0.8, which indicates that transmission on this tone is virtually swollen by crosstalk. Fig. 2(a) illustrates this.

What is done in practice in 2SB is to establish an acceptable segment in the CDR line (Fig. 2(b)). This acceptable segment shall be defined as varying according to a normal function (many options are possible), characterized by a center l^k and a variance v. It is the role of the SMC to establish these two variables. For doing this, it needs to have access to the w_n values of each user. The segment of acceptable crosstalk can be interpreted as the amount of damage from crosstalk that a given user can stand for a globally good solution. The center of it, l^k, is the ideal level. In the scenario of interest, all users except the N-th receive significant levels of crosstalk. The sequence of iterations in 2SB will have the SMC adjusting the VLs for all users as to make the CDR of crosstalk victims to be equal or in the proximities of l^k.

The amount of values sent by one modem is $K + 1$: K values for the CDR, one for each tone, and one for the w_n. The next section will illustrate the processing in the SMC.

C. SMC Processing

The role of the SMC consists of two things: one, to process crosstalk data contained in the CDR for adapting VL parameters of all lines so that they more faithfully represent the real crosstalk environment; and, two, to establish for each tone what is l^k, the center of the acceptable crosstalk segment. Both will be detailed in the next subsections. It will be seen that CDR should be "filtered" by a binary $N \times N$ matrix in which the elements indicate whether there is assumed interference between any two users; and that l^k should be chosen as the second greatest w_n of all users that allocate power on tone k.

1) CDR Processing: Processing of the information contained in the CDR is demonstrated in steps 8 to 15 in Alg. 1. In step 10, CDR_j^k of all $j \neq n$ should be multiplied by w_j and c_{nj}. The greatest value should be stored in the variable *maximum* CDR, mCDR_n^k, that represents the damage user n causes to its greatest victim. The variable c_{nj} is an element of $\mathbf{C}_{(N \times N)}$, the *crosstalk matrix*, a binary matrix in which element c_{nj} should present 1 if it is considered that transmitter n interferes considerably with receiver j; and 0 if there is no such considerable crosstalk. For the scenario of interest,

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 1 & \cdots & 1 & 0 \end{bmatrix}, \qquad (7)$$

which indicates that the crosstalk channels to worry about are from user n to all $j < n$. The $\mathbf{C}$ matrix is not built in a per-tone basis. It is built for the network.

The objective of $\mathbf{C}$ is to locate crosstalkers and punish them accordingly. Consider a simple example. In the topology in Fig. 1, consider that user 2 reports a high value of CDR on a given tone. Column 2 of $\mathbf{C}$ should reflect the fact that users 3 to N can contribute significantly for such high CDR. User 1 is free of any blame and should be exempted from punishment. Punishment comes in the form of an increase in the VL parameter $\tilde{h}_n^k$ for all $n > 2$, while user 1 should have it decreased. In the following iteration, users that contribute to such crosstalk will be more restricted at allocating power on this tone, so that user 2 should feel less crosstalk, and user 1 will be freer in allocating power.

This simple kind of channel information demanded draws a line between 2SB and previous optimal or near-optimal solutions. The requirement of reliable per-tone crosstalk channel information was mandatory for most DSM schemes cited in Section II. In the simulation section in this work, $\mathbf{C}$ is set through simple inspection of the network. In practical situations more sophisticated and reliable heuristics can be applied. The point to be stressed is that obtaining $\mathbf{C}$, no matter what method is used, should be much simpler to obtain than estimating the channel completely[1].

2SB will adapt the VL parameters of each user (lines 12 to 15 in Alg. 1) according to the comparison of mCDR_n^k with the ideal level of crosstalk. It should continue until damage from crosstalk, weighted by mCDR, is close enough to l^k for all users and tones.

The variance v of the normal curve of acceptable crosstalk is a constant value which should be available to the SMC from the start. It has been observed from experiments that this value does not have great influence on the final PSD design.

2) Choosing l^k: From simulation experience, it is known that every good DSM solution has its modems always operating in a high SNR regime [8]. Ideally, crosstalk should always be much lower than background noise. Unfortunately, this is not practical, and modems should sustain some level of crosstalk. The question of interest is to find what is this level. For example, for scenario in Fig. 1, 2SB should find the level of crosstalk that user 1 can sustain from all other users for a globally good solution.

Considering the approximation of high SNR regime to be desirable and that the greatest interferer for the transmission of user 1 is user N, the level of crosstalk l^k which should be sustained by user 1 for a globally good solution can be approximated by

$$l^k = \frac{w_N}{\log_2\left(\frac{p_1^k}{\Gamma\sigma_1^k}\right)} \times \frac{|h_{N1}^k|^2}{\tilde{h}_N^k}. \qquad (8)$$

See the appendix for the demonstration.

The right-hand part of the multiplication above is a measure of the difference between the actual crosstalk channel from user N to user 1 and the virtual crosstalk channel between user N and its VL. In an ideal case, the sequence of iterations will have the SMC suggesting values for $\tilde{h}_N^k$ such that the discrepancy to the real crosstalk channel is every time smaller. The approximation that, on the long run of iterations, the difference between the crosstalk channels tends to be of less importance will be considered. The quotient will thus be considered 1.

The denominator of the left-hand part in Eq. 8 is a measure of channel quality. Channels in which user 1 allocates more bits should be considered more carefully, i.e. their level of acceptable crosstalk should be smaller. Nevertheless this makes perfectly good intuitive sense, it was observed in experiments that such measure of channel quality does not play so important a part for achieving a globally good solution. Taking this fact under consideration, the choice of l^k is simplified to

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[1] Instead of crosstalk channel estimation, the method relies on estimation of background noise for calculating CDR and setting background noise for the VLs. The authors would like to claim that this is a small price to pay for not having to estimate crosstalk channels. This is so for two reasons. One, estimating background noise is much simpler than crosstalk channels. In the time of writing, there is still no standardized method for crosstalk channel estimation. And, two, the sheer size of the task: a network of 20 users has 20 different background noise patterns, but 380 crosstalk channels.

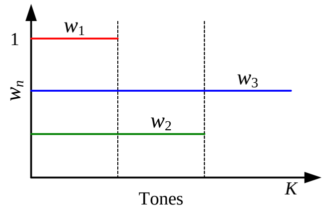

Fig. 3. For $w_3 > w_2$.

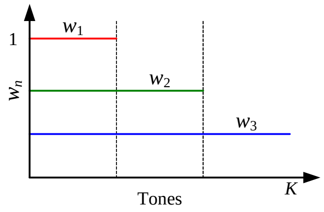

Fig. 4. For $w_3 < w_2$.

$$l^k = w_N, \tag{9}$$

which indicates that the level of crosstalk that user 1 should sustain is directly proportional to the emphasis over the VL required for user N to achieve its minimum rate. The more emphasis user N needs to have in respect to its VL, the more crosstalk user 1 has to withstand.

Two more problems need to answered for coming up with the l^k level. First, we should not rely on the approximation that one given user is the greatest interferer to user 1. Such identification depends heavily on network knowledge and should be avoided. For the scenario of interest, the question would be which w_n should be chosen for l^k in tones where all users allocate power. Second, there can be cases in which a user with a longer line may stop transmission in higher frequencies due to poor direct channel conditions. In such tones, where only a subset of users allocate power, what is the level of crosstalk the remaining users are allowed to cause to each other?

About the first issue, the solution is quite simple. The greatest w_n of all users except the first should be chosen. The greater the w_n, the greater the priority user n needs over its VL to achieve its $R_n^{\min}$. Since $w_1 = 1$, the second greatest values of w_n should be chosen.

The second problem will be solved with the aid of Figs. 3 and 4. For simplicity, a 3-user scenario will be temporarily considered. Extensions to N user scenarios are straightforward. The two pictures show the tones in which users allocate power and their respective w_n. As frequencies increase, users with longer lines stop transmission due to poor direct channel conditions. In frequencies where only users 2 and 3 transmit, the second greatest value of w_n should again be chosen for l^k. For understanding this, bear in mind that analysis of the topology leads us to conclude that the only significant crosstalk channel is from user 3 to user 2.

Fig. 3 depicts a situation in which w_3 is greater than w_2. Hence $l^k = w_2$. The mCDR for users 2 and 3 in tones where only these users allocate power will be calculated as

$$\begin{aligned} \text{mCDR}_2^k &= \max\left(c_{21}w_1\text{CDR}_1^k\ ,\ c_{23}w_3\text{CDR}_3^k\right) && (10)\\ &= \max(0,0) = 0, \\ \text{mCDR}_3^k &= \max\left(c_{31}w_1\text{CDR}_1^k\ ,\ c_{32}w_2\text{CDR}_2^k\right) && (11)\\ &= \max(0, w_2\text{CDR}_2^k) = w_2\text{CDR}_2^k. \end{aligned}$$

As can be seen, the variable mCDR_3^k is upper bounded by w_2. Since $l^k = w_2$, the VL for user 3 will be always decreased. User 3 is allowed to cause any level of damage to user 2. If user 2 starts to have difficulty in achieving its minimum data rate due to crosstalk from user 3, this will be reflected in an increase in the priority over its VL (i.e greater w_2).

The case in which $w_2 > w_3$ will be demonstrated with the aid of Fig. 4. For such a situation, $l^k = w_3$, the lower value. The mCDR_n^k will be calculated in the same way as in Eq. 10 and 11. Since mCDR_3^k is upper bounded by w_2, user 3 is allowed to cause a w_3/w_2 damage in user 2. Say, for instance, that w_3 and w_2 are equal to 0.25 and 0.5, respectively, then user 3 is allowed to cause 0.5 decrease in bit loading for user 2.

In summary, what is basically proposed is to break down the interaction of the N users in the network to pairs. For each tone, 2SB chooses which is the greatest victim of crosstalk (call it V) and identifies which are its potential interferers. Then the algorithm groups those interferers together into a superuser (thus forming a pair with V), which should be represented by the user that needs most priority over its VL, i.e. the user that needs the biggest portion of channel resources (call it X). If the superuser is causing an acceptable amount of crosstalk to V, then so does X and all the other less important crosstalkers. The approximation made is that the group can be fairly punished when represented by X. Section IV will present evidence that this approximation is fairly good and yields close-to-optimal rate regions.

In frequencies where only one user allocates power, it should not be constrained by any means and thus l^k should be chosen as zero. This will always produce a decrease in its VL, to the point where it almost disappears.

D. Message-passing 2

The last step of one iteration of 2SB is the reporting of updated VL parameters to all modems. The message is of size K for each modem.

IV. SIMULATION RESULTS

The proposed algorithm will be compared with previous solutions in the scenario of Fig. 1 with $N = 3$ and $l_1 = 5$km; d_2 and l_2 equal to 2 and 4 km, respectively; and d_3 and l_3 equal to 4 and 3 km.

We are interested in downstream ADSL. It is used 0.5 mm cables (AWG 24 cable) and it is desired BER of 10^{-7}, noise margin of 6 dB and coding gain of 3 dB, which results in a Shannon gap of 12.8 dB. The values for Δ_f and f_s are set to 4.3125 kHz and 4 kHz, respectively. Modems have at their disposal a maximum power of 20.4 dBm [9]. In each line it is included noise model ANSI A [10].

Fig. 5 depicts the rate regions for ASB, ISB, OSB, SCALE, IWF and the proposed 2SB. It is desired to have user 2 operating at 2 Mbps for all points in the curves. 2SB has interference matrix set as in Eq. 7. For ASB, reference line parameters were always set to match the crosstalk environment

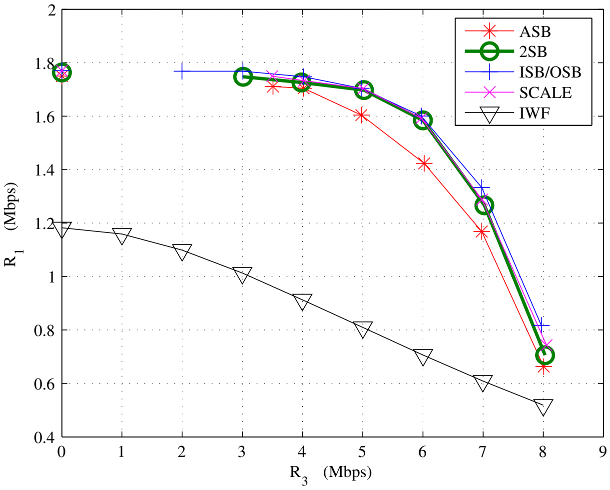

Fig. 5. Rate region for the scenario cited.

to user 1. It can be seen that 2SB performs very close to optimal but with significant less demanding logistics.

In this specific scenario, 2SB converges in about 15 iterations. For plotting Fig. 5, the initial value for VL crosstalk channels was set to -45 dB and the variance v of the normal function to 0.15. These are fairly unimportant values for results and convergence. One parameter to which results depend highly is the setting of the crosstalk matrix.

There is not yet proof of convergence or optimality for the algorithm, but to this point tens of scenarios have been tried (not shown due to space limitation) and all experiments were as successful as the one presented.

V. FINAL REMARKS

This text presented 2SB, a new method for power allocation for DSL. 2SB can be classified according to the parameters laid out in Section II as achieving a good rate region with low computational cost, semi-centralization and requiring only very simple crosstalk channel knowledge. This makes it a strong competitor among other state of the art algorithms.

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APPENDIX

We want to find ideal level of crosstalk l^k user 1 should sustain. It is considered that user N is the only considerable crosstalker in this case, all other users' interference being negligible. Hence,

$$\begin{aligned} l^k &= \mathrm{CDR}_1^k = 1 - \frac{b_1^k(\mathrm{xt}+\sigma)}{b_1^k(\sigma)} \\ &= 1 - \frac{\log_2\left[1 + \frac{p_1^k}{\Gamma(\sigma_1^k + |h_{N1}^k|^2 p_N^k)}\right]}{\log_2\left(1 + \frac{p_1^k}{\Gamma\sigma_1^k}\right)} \end{aligned} \tag{12}$$

From simulation experience, it is know that in every good DSM solution users operate in a high SNR regime,

$$p_1^k >> \sigma_1^k >> |h_{N1}^k|^2 p_N^k, \tag{13}$$

so that the following approximation is made

$$\log_2\left[1 + \frac{p_1^k}{\Gamma\left(\sigma_1^k + p_N^k h_{N1}^k\right)}\right] \approx \log_2\left(\frac{p_1^k}{\Gamma\sigma_1^k}\right) - \frac{p_N^k h_{N1}^k}{\sigma_1^k}, \tag{14}$$

and l^k can be rewritten as

$$l^k \approx \frac{p_N^k |h_{N1}^k|^2}{\sigma_1^k} \frac{1}{\log_2\left(\frac{p_1^k}{\Gamma\sigma_1^k}\right)}.$$

Under the high SNR approximation, power allocation for user N when optimizing against a VL can be simplified [8]:

$$l^k \approx \left[\frac{w_N}{\lambda_N + \frac{\tilde{h}_N^k}{\tilde{\sigma}_N^k}} - \sigma_N^k\right] \frac{|h_{N1}^k|^2}{\sigma_1^k} \frac{1}{\log_2\left(\frac{p_1^k}{\Gamma\sigma_1^k}\right)}. \tag{15}$$

The left-hand term of the expression between brackets can have λ_N disregarded. The right-hand term in the brackets, σ_N^k, will be close to zero when multiplied by the expression outside the brackets because it presents $|h_{N1}^k|^2$. This value is negligible, since it was normalized by the direct channel of user N, $|h_{NN}^k|^2$ (Section II). Eq. 15 is then simplified to

$$l^k \approx \left[\frac{w_N \tilde{\sigma}_N^k}{\tilde{h}_N^k}\right] \frac{|h_{N1}^k|^2}{\sigma_1^k} \frac{1}{\log_2\left(\frac{p_1^k}{\Gamma\sigma_1^k}\right)}. \tag{16}$$

The terms for background noise are roughly the same and can be ignored. Eq. 8 follows. ■

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