Low-Complex ICI Cancellation for Improving Doppler Performance in OFDM Systems

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Abstract—A combination of receiver windowing and inter-carrier interference (ICI) canceling is proposed for improving the Doppler performance of Orthogonal Frequency Division Multiplex (OFDM) systems. The windowing reduces the ICI, and also reduces the required number of canceled bins. The effect of windowing is analytically derived, and bounds are given on the possible gain that can be obtained. Since successful ICI cancellation relies on that the channel can be accurately estimated, the paper also give examples for how different one-dimensional and two-dimensional channel estimators can be expected to work for various Doppler spread and delay spread of the channel. To verify that the proposed approach for ICI canceling works, as well as studying the impact of channel estimation errors, simulations are performed using the key parameters of DVB-H. As a result it is found that with proper channel estimation, windowing but no ICI cancelation can increase the allowed Doppler by 20 %, ICI cancellation but no windowing can also increase the allowed Doppler by 20 %, and if both windowing and ICI cancelation are used, the allowed Doppler can increase by more than 40 %.

I. INTRODUCTION

A major technical problem for a system based on Orthogonal Frequency Division Multiplex (OFDM), is its susceptibility to frequency offsets, phase noise, and Doppler in a time-varying channel. This is because inter-carrier interference (ICI) between the OFDM sub-carriers results, and greater difficulty in the channel estimation.

The ICI can easily be reduced by increasing the sub-carrier spacing, i.e., making the OFDM symbols shorter. However, since the cyclic prefix (CP) typically used in OFDM systems needs to be long enough to guarantee that no inter-symbol interference (ISI) is experienced, the improved Doppler performance comes at a cost of increased overhead. Especially for broadcast applications where so-called single frequency networks (SFN) are used, the CP must be large.

To date most OFDM systems have targeted fixed or low mobility applications. More recently, however, high mobility services have been targeted; examples being DAB, DVB-H, and the evolution of 3G. This work relates to improving the performance of OFDM in Doppler Channels, with emphasis on DVB applications.

That ICI cancellaton is a differentiator when it comes to DVB-H is readily seen in the implementation guidelines of the DVB-H standard [2], where the performance for two kinds of reference receivers is described - A Typical one which does not use ICI cancellation and a Possible one which relies on ICI cancellation.

ICI cancelation in relation to DVB has been considered by others, e.g. [5],[12]. The proposed schemes for ICI cancellation are relatively complex compared to the other baseband processing in the receiver since it usually is based on forming a matrix of coefficients describing the ICI coupling into adjacent channels. In [12], the complexity issue of the ICI cancellaton was addressed by taking an iterative approach. However, with the complexity measured by the silicon area needed for the implementation, the complexity was still significant. The approach taken in this paper has been found to increase the area of the digital baseband by a relative moderate amount.

Another means to reduce the impact of e.g. frequency offset in OFDM systems is by using windowing [9] and [10].

In this paper, a low complexity form of ICI reduction is described. The technique uses an adaptive receiver window, to improve performance and reduce the required number of terms to cancel in case of low delay spread channels.

The remaining part of the paper is organized as follows. Section II gives the theory, especially the impact of windowing is derived and bounds on the gain that can be obtained with ICI cancellation are given. The impact of channel estimation error and different approaches for channel estimation are presented in Section III. Simulation results for a DVB-T/H system are presented in Section IV, and, finally, conclusions are drawn in Section V.

II. THEORY

In OFDM, an IFFT is used at the transmitter side and an FFT at the receiver side. The sent sequence is formed by first taking the IFFT, and then prepending the last part. The prepended part is known as the CP or the guard interval (GI). Specifically, first

$$s_n = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{j2\pi kn/N} \quad n = 0, 1, \ldots, N - 1 \quad (1)$$

is generated, and then the GI is obtained as

$$s_{-n} = s_{N-n}, \quad n = 1, 2, \ldots, N_G, \quad (2)$$

where $N_G$ is the number of samples in the GI.
A. ICI due to Doppler - Rectangular Windowing

At the receiver side, an ISI free portion of the transmitted signal is found and then demodulated by an FFT. First, suppose that this is done by using a rectangular window of length \( N \). This is the standard approach, and henceforth we will sometimes refer to this as the case when no windowing is used. Let the received sequence obtained in this way and input to the FFT be denoted \( r_0, r_1, \cdots, r_{N-1} \). Let \( H_k, n \) denote the transfer function of the channel for carrier \( k \) and sample \( n \) of the sent signal. Considering the \( K \)th bin at the output of the FFT, this equals

\[
R_K = \sum_{n=0}^{N-1} r_n e^{-j \frac{2\pi}{N} K n} = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} S_k H_{k,n} e^{j \frac{2\pi}{N} (k-K)n}
\]

\[
= \sum_{k=0}^{N-1} S_k \sum_{n=0}^{N-1} \frac{1}{N} H_{k,n} e^{j \frac{2\pi}{N} (k-K)n}.
\]

(3)

In case the channel is static, i.e., \( H_{k,n} = H_k \) for all \( n \), then

\[
R_K = \sum_{k=0}^{N-1} S_k H_k \sum_{n=0}^{N-1} \frac{1}{N} e^{j 2\pi (k-K)n/N} = S_K H_K.
\]

(4)

Now, suppose that the channel for the \( K \)th carrier during the reception of one OFDM symbol is assumed to vary in a linear fashion. Specifically, let

\[
H_{K,n} = \Pi_K + \left( n - \frac{N-1}{2} \right) \frac{H'_K}{N},
\]

(5)

where \( H_{K,n} \) is the channel change during the (information part of the) symbol and \( \Pi_K \) is the average channel experienced during the OFDM symbol. Furthermore, let \( R_{i,j} \) denote how the \( j \)th symbol \( S_j \) affects the \( i \)th output. It then follows that

\[
R_{K,K} = S_K \frac{1}{N} \sum_{n=0}^{N-1} \Pi_K = \Pi_K S_K
\]

(6)

and

\[
R_{K,K+L} = S_{K+L} \frac{1}{N} \sum_{n=0}^{N-1} H_{K+L,n} e^{j \frac{2\pi}{N} nL}
\]

\[
= S_{K+L} H'_{K+L} \frac{1}{N} \sum_{n=0}^{N-1} \frac{n}{N} e^{j \frac{2\pi}{N} nL},
\]

(7)

where the last step follows from that the different tones are orthogonal if the channel is static. To proceed, we use that if \( N \) becomes large then

\[
\sum_{n=0}^{N-1} \frac{n}{N} e^{j \frac{2\pi}{N} nL} \approx \int_0^1 te^{j 2\pi t L} dt = \frac{1}{j 2\pi L}.
\]

(8)

Consequently, the ICI on carrier \( K \) caused by the symbol sent on carrier \( K + L \) is given by

\[
R_{K,K+L} \approx S_{K+L} H'_{K+L} \frac{1}{j 2\pi L} = S_{K+L} H'_{K+L} G_L.
\]

(9)

Henceforth, we will refer to \( G_L \) as the leakage coefficient. Clearly, it would be desirable to make this as small as possible.

A schematic figure for how windowing and ICI cancellation might be implemented is shown in Figure 1.

From (9) it is also straight-forward to derive the total degradation caused by ICI using the statistics of \( H' \) and summing over all \( L \). In case Jakes’ model is assumed for the Doppler spectrum [7], the well-known formula for the ICI power caused by Doppler is obtained [8]

\[
P_{ICI} = \frac{\pi^2}{6} f_d^2,
\]

(10)

where \( f_d = f_D / \Delta f \) is the normalized Doppler, i.e., the actual Doppler \( f_D \) divided by the sub-carrier spacing \( \Delta f \).

B. ICI due to Doppler - Non-Rectangular Windowing

In many practical situations, the excess delay of the experienced channel is considerably smaller than the duration of the used GI. One such situation is broadcast, like the newly developed DVB-H standard, where the GI must be chosen for worst case delay spread within the coverage area, but where some users might experience considerably smaller delay spread. If this is the situation, then the part of the GI not affected by ISI might be used to improve the performance of the receiver by applying a non-rectangular window function, \( w(n) \).

Henceforth, let \( W \) denote the number of samples from the GI that is used in this window. Clearly \( 0 \leq W \leq N_G \). We restrict our attention to windows that preserve the orthogonality between the carriers, i.e., Nyquist windows.

When windowing is used, a part of the GI not affected by ISI is after weighting added to the corresponding part of the information part of the symbol. Before considering a specific window, suppose that \( r_{N-W}, r_{N-W+1}, \cdots, r_{N-1} \) are replaced by \( r_{-W}, r_{-W+1}, \cdots, r_{-1} \) before applying the FFT. It is straight-forward to show that the ICI term \( R_{K,K+L} \) in this case becomes

\[
R_{K,K+L} = S_{K+L} H'_{K+L} e^{-j \frac{2\pi}{N} W L} / j 2\pi L.
\]

(11)

Next, consider using a window function where \( W \) samples from the GI are combined with the corresponding part at the end of the OFDM symbol by multiplying each of these...
samples by 0.5 and then add them together. This window can be expressed as:

\[ w(n) = \begin{cases} 
0.5 & : -W \leq n \leq -1 \\
1.0 & : 0 \leq n \leq N - 1 - W \\
0.5 & : N - W \leq n \leq N - 1.
\end{cases} \]

Since this can be viewed as an average of (9) and (11), we obtain

\[
R_{K,K+L} = S_{K+L}H_{K+L}' + \frac{1}{2\pi L} e^{-j\frac{\pi}{2}W L} \cos \left( \frac{\pi L}{N} \right) W. \tag{12}
\]

The factor \( e^{-j\frac{\pi}{2}W L} \) is merely a phase shift caused by how the folding is done. This phase shift can be avoided by having a symmetric window with respect to the FFT position, i.e., by letting the first sample that is fed to the FFT be \( W/2 \) samples prior to \( r_0 \).

Comparing (9) and (12), it is readily seen that the simple window above reduces the ICI power from the \( L \)th bin by a factor \( \cos^2 \left( \frac{\pi}{2} W L \right) \).

Implementation-wise, the simple window described above is easily implemented since the values of the window is 0, 0.5, or 1, and multiplication with 0.5 is obtained by a one step right shift of the digital value. Below we refer to this window as a 2-step window. Other window functions that also are easy to implement is when the window takes the values 0, 0.25, 0.5, or 1. Below these windows are referred to as a 4-step windows.

In Tables I and II, the leakage coefficients for different sub-carrier offsets, \( L \), and different window lengths are shown for the 2-step windows and the 4-step windows, respectively. The window length is given relative to the FFT size, \( N \). For instance, if the length of the GI is \( N/4 \), and \( W = 3N/16 \), this means that 75% of the GI is used for the window. The first row (\( W = 0 \)) corresponds to that a rectangular window is used. As can be seen, the leakage coefficients are reduced, especially as \( L \) is increased.

### TABLE I

<table>
<thead>
<tr>
<th>( L \times W )</th>
<th>( 2b ) in ( 4b ) in ( 6b ) in</th>
<th>dB</th>
<th>dB</th>
<th>dB</th>
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<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.500</td>
<td>0.333</td>
<td>0.250</td>
</tr>
<tr>
<td>1/16N</td>
<td>0.000</td>
<td>0.500</td>
<td>0.333</td>
<td>0.250</td>
</tr>
<tr>
<td>2/16N</td>
<td>0.984</td>
<td>0.462</td>
<td>0.273</td>
<td>0.177</td>
</tr>
<tr>
<td>3/16N</td>
<td>0.924</td>
<td>0.354</td>
<td>0.128</td>
<td>0.000</td>
</tr>
<tr>
<td>4/16N</td>
<td>0.832</td>
<td>0.191</td>
<td>-0.065</td>
<td>-0.177</td>
</tr>
<tr>
<td>5/16N</td>
<td>0.707</td>
<td>0.000</td>
<td>-0.236</td>
<td>-0.250</td>
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### TABLE II

<table>
<thead>
<tr>
<th>( L \times W )</th>
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<th>dB</th>
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### TABLE III

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<tr>
<th>( W \times 2b ) in ( 4b ) in</th>
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<th>dB</th>
<th>dB</th>
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<td>8.75</td>
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### TABLE IV

<table>
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<th>dB</th>
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</thead>
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<td>0.250</td>
<td>0.302</td>
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</tr>
<tr>
<td>1/16N</td>
<td>0.000</td>
<td>0.250</td>
<td>0.302</td>
<td>0.354</td>
</tr>
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C. Bounds on the Achievable Performance Gains

If the statistics in terms of the sent data and the channel variations are the same for all sub-carriers, the average power of the ICI can be written

\[
P_{ICI} \sim f_D^2 \sum_{L \neq 0} |G_L|^2. \tag{13}
\]

Thus, the potential gain depends on how much the sum above can be reduced. In case of no windowing and no ICI cancellation, the sum equals [6]

\[
\sum_{L \neq 0} |G_L|^2 = \sum_{\substack{L \neq 0 \\
L \neq 0}} \frac{1}{L^2} = \frac{\pi^2}{3}. \tag{14}
\]

When ICI cancellation is used, the total power is calculated by removing the corresponding terms from (13). The results are presented in Tables III and IV for 2-step windows and 4-step windows, respectively. Canceling 2 bins means that ICI from the two closest bins (one on each side, i.e., \( L = \pm 1 \)) are canceled, and canceling 4 bins means that ICI from the four closest bins (two on each side, i.e., \( L = \pm 1, \pm 2 \)) are canceled, etc.

Since the ICI is proportional to \( f_D^2 \), see (13), the corresponding increase in what Doppler can be handled is easily obtained. For instance, in case of \( W = 3N/16 \) and no canceling \( f_D \) can be increased by about 35%, and in case the ICI from the two closest carriers was perfectly cancelled, then the maximum Doppler would be improved by almost a factor of three.

### III. Channel Estimation Errors

In case of high Doppler, it is not only ICI that becomes a problem. In addition, channel estimation becomes harder. Since successful ICI cancellation relies on accurate estimation of both the sent data, \( S_{K+L} \) and the channel variation \( H_{K+L}' \), see (9), one can expect that the ideal gain might be hard to obtain in practice. Since ICI as such worsen the channel estimation, it is readily seen that an iterative approach where
channel estimation and ICI cancellation are successively improved might be a feasible approach. This is an approach taken in, e.g., [5].

An iterative approach would also benefit from non-rectangular windowing, since the reduced ICI would allow for faster convergence. In this work, however, where low complexity is emphasized, this is not considered. Here it is assumed that channel estimation is performed first, followed by ICI cancellation.

To get a bound on how well the channel estimation can be performed, consider the noise-less case where the error in the channel estimation is due to the interpolation error only. This is plotted in Figure 2 for the DVB system using different set of pilot patterns for channel estimation. In DVB, the pilots occur in every third frequency bin, and every fourth symbol in a staggered format, marked with crosses in the figure. In Figure 2, the performance is depicted as a function of normalized Doppler and delay spread for different pilot combinations. Refer to text for detailed description.

![Fig. 2. The channel estimation interpolation limit (noiseless) as a function of normalized Doppler and delay spread for different pilot combinations. Refer to text for detailed description.](image)

The Doppler performance can be improved if pilots in the frequency direction are included (mid/bottom Figure 2). Since they are offset in time they effectively can lead to an increase in the Nyquist rate when the delay spread is small. The complexity of the channel estimation is reduced by only selecting the pilots that really have an impact on the channel estimation, and reducing the memory requirements by reducing the number of future coefficients. The bottom plot uses 8 coefficients (instead of 4) and reduces forward memory requirements to 3 symbols (instead of 7). As can be seen, it has excellent performance at low delay spreads.

IV. SIMULATION RESULTS

As discussed in the previous sections, successful ICI cancellation requires that the channel as well as its derivative can be estimated sufficiently well. To evaluate how well this requirement can be met in an actual system, simulations were performed for the 8k mode in DVB-H, [1], [4] when the largest GI interval was used. This means that the carrier spacing is \( \Delta f = 1116 \) Hz. Furthermore, if the channel estimation is performed by first interpolating in the time direction, the Nyquist rate equals 112 Hz (normalized Doppler \( f_d = 0.1 \)).

The reason for performing channel estimation by first interpolating in time and then in frequency is to allow for larger delay spread to be handled. However, in case the delay spread of the channel is small, which is exactly the case when it is feasible to apply a non-rectangular window, then the channel estimation can be done in the frequency direction directly.

The forward error correction (FEC) coding used in DVB-H consists of the concatenated coding scheme used in DVB-T [3] extended with another layer of coding on the link layer, which gives time-diversity of some 200 ms under typical operating conditions. To evaluate the gain that can be obtained by windowing and ICI cancellation, the byte error rate (BER) at the output of the concatenated code was used. The operating point where the link layer FEC code in DVB-H will give error
free reception depends of course on the statistics of the errors out from the concatenated code as well as on the code rate used by the link layer code.

If the errors out from the concatenated code is assumed to be spread uniformly by the interleaver used by the link-layer code, a BER of a few percent from the RS decoder will results in acceptable performance after the link-layer FEC. The horizontal line at 6.1% BER in the figures corresponds to the required BER in case of the lowest rate of the link-layer code.

All simulations are performed using the Typical Urban channel model (TU6), [2], [11] for 16-QAM and rate 2/3 of the convolutional code. Also, in all simulations where ICI canceling is applied, only the ICI from the two adjacent bins are canceled, i.e., $L = \pm 1$. The simulations are performed at large signal-to-noise ratio, so errors are due to channel estimation and/or ICI caused by Doppler, rather than noise.

In Figure 3, the performance is shown when the channel estimation is done in the time direction first and no windowing is used. The interpolation filter is a Wiener filter with six taps, and it is readily seen that channel estimation rather than ICI is the problem. (Recall that the Nyquist frequency was 112 Hz.) Since ICI is not the problem, applying a window does in this case only give a marginal gain.

If instead the channel estimation is performed by first interpolation in the frequency direction, the “Nyquist-barrier” posed by the interpolation filter in the time direction can be passed. In Figure 4, the performance obtained without windowing is depicted with and without ICI cancellation. As can be seen, ICI canceling increases the acceptable Doppler by roughly 20%. If windowing is used, as in Figure 5, we see that just applying a window is in this case as effective as ICI cancellation without windowing. Finally, if both windowing and ICI cancelation are used, the Doppler that can be handled increases by about 40%.

V. CONCLUSION

Low complex ICI cancellation for OFDM systems was considered in this paper. It was shown analytically as well as by means of simulations that windowing is an effective means to improve the Doppler performance, in particular in combination with ICI cancellation. Bounds on the possible gain that can be obtained were derived as a function of the shape of the window that is used as well as on the number of terms that are canceled. It was also discussed that channel estimation typically will limit the gain that can be achieved in practice, at least if low complexity is mandated for the ICI cancellation scheme. Finally, simulation results for a DVB-H system were presented. It was seen that if the channel estimation is done by first interpolating in time, then channel estimation rather than ICI will limit the performance. Therefore, to obtain good Doppler performance, channel estimation was done by interpolating in the frequency direction directly. With this approach, it was found that windowing and ICI cancelation can increase the Doppler that can be handled by as much as 40%.

REFERENCES

[3] ETSI EN 300 744 V.1.4.1 (2001-01), Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television.