Abstract—In this work the modeling and capacity of a dual polarized (DP) MIMO channel is addressed. The modeling includes channel parameters such as receive and transmit correlation, channel cross-polarization discrimination (XPD), and antenna parameters such as polarization state, polarization parallelity and coupling. The capacity of the DP MIMO channel is evaluated and compared to the capacity of a single polarized (SP) MIMO system. The SP MIMO system with spatially separated antenna sensors has the advantage that it also offers array gain and will therefore during idealized conditions outperform the DP MIMO system. However, in this work it is found that this advantage often is reduced and sometimes even lost when channel and antenna imperfections such as, e.g., correlation, channel XPD, and polarization mismatch are introduced. The main conclusion is that DP antennas might not always yield the best MIMO performance, but instead offer compact antenna solutions for mobile devices and robust performance that is more insensitive to the aforementioned imperfections.

I. INTRODUCTION

The use of multiple and spatially separated (SS) antennas at both transmit and receive side offers several possibilities such as, e.g., spatial multiplexing and spatial diversity. However, data multiplexing and diversity can also be achieved in the polarization domain. Measurements have shown that vertically and horizontally polarized electromagnetic (EM) waves in many non-line-of-sight (NLOS) scenarios fade almost independently (with some degree of cross-coupling), and in line-of-sight (LOS) scenarios two transmitted orthogonal polarizations remain orthogonal through the channel, e.g., [1]. With this observation in mind one can, thus, construct two sub channels in the polarization domain by using a dual polarized (DP) antenna at both transmit and receive sides. An advantage in using DP antennas is that the two polarizations can be colocated which offers a compact antenna realization, while SP antennas need to have a larger spatial separation to yield uncorrelated channels. One can of course also combine the polarization and spatial domains by using spatially separated dual polarized (SS-DP) antennas. However unless there is a polarization mismatch, spatially separated single polarized (SS-SP) antennas benefit from array gain.

Several measurement campaigns and channel models have been reported in the literature. In [2] it is reported that vertically polarized signals transmitted over urban environments with sufficient scattering will also have a strong received horizontally polarized component, which if exploited by the receive antenna system leads to a significant diversity gain. In [3] a more advanced geometrical scattering model for DP MIMO is described and in [4] and [5] the modeling and capacity of \(2 \times 2\) DP multiple-input multiple-output (MIMO) system is studied. It is found that a Ricean channel model can successfully describe the polarized MIMO channel and that the \(2 \times 2\) DP MIMO system offer full multiplexing gain for high K-factors. It is concluded in [1] that vertically and horizontally polarized waves on the average experience the same path loss and that all the co-polarized and cross-polarized envelope correlation coefficients are small in both indoor and outdoor measurements. Furthermore, it is concluded that the channel cross-polarization discrimination (XPD) is large (meaning good isolation between the vertical and horizontal polarizations) for many scenarios. In fact, the channel XPD varied between 5 to 15 dB with being the largest for outdoor LOS-like channels and smallest for more scattering NLOS channels (both indoor and outdoor).

The general conception is often that MIMO systems with SS-SP antennas thanks to array gain offer higher communication performance than systems equipped with DP antennas. But, what happens when we infer imperfections in the transmitter/receiver antennas and channel? For example, an EM wave can change its polarization state during channel propagation and, hence, a SP receive antenna might become mismatched to the incoming wave. Even a simple rotation of a mobile device might cause polarization mismatch. Furthermore, we need to identify important parameters and find out what parameters influence the performance most. These parameter choices are important input to system and antenna designers. In fact, multiple DP and SP antennas are strong candidates to be used in future communication systems such as, e.g., long term evolution (LTE) systems. For example, the measurements reported in [6] show that the use of multiple DP base station antennas yields good communication performance.

To understand the benefits and drawbacks from using DP antennas compared to SS-SP antennas for MIMO transmission, we need to model the overall polarized MIMO channel including the channel itself and the antennas that are used to sense the channel. In this work, we model the channel medium with a Ricean channel model that is described in vertical (V) and horizontal (H) polarizations. Since any polarization can be described as a linear combination of V and H (they form a basis) we can then calculate the channel response to any polarization that we like. This way the medium is kept separate from the antennas that are used to access the channel. Furthermore, we use a channel normalization that can capture the effect of polarization mismatch. Using the channel and
antenna models we evaluate the capacity for DP systems and compare it to SS-SP systems for various parameter settings. We also show how parameters such as correlation, channel XPD, Ricean K-factor, polarization parallelly, and coupling affect the performance.

II. CHANNEL MODELING

As in, e.g., [7], the DP MIMO channel is modeled as a Ricean fading channel, i.e., the channel matrix can be composed of a fixed (possibly line-of-sight) part and a random (fast fading) part according to

$$
H = \sqrt{\frac{K}{K+1}} \tilde{H} + \sqrt{\frac{1}{K+1}} H,
$$

(2.1)

where $K$ is the Ricean K-factor, $\tilde{H}$ is a deterministic matrix and $H$ is a random matrix. The random matrix $H$ consists of complex Gaussian entries which are independent from one channel realization to the next. In other words, if $K = 0$ then $H$ models a Rayleigh fading channel and if $K \to \infty$ it models a static channel. The channel matrix is described in V and H polarizations, i.e., its elements represent the input-output relation from V to V, V to H, H to H, and H to V polarized waves, and a $4 \times 4$ MIMO channel realized by having two spatially separated DP antennas on each side can be written as

$$
H = \begin{bmatrix}
h_{1V,1V} & h_{1V,1H} & h_{1V,2V} & h_{1V,2H} \\
h_{1H,1V} & h_{1H,1H} & h_{1H,2V} & h_{1H,2H} \\
h_{2V,1V} & h_{2V,1H} & h_{2V,2V} & h_{2V,2H} \\
h_{2H,1V} & h_{2H,1H} & h_{2H,2V} & h_{2H,2H}
\end{bmatrix} = \begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
$$

(2.2)

where the scalar channel between the $i$th transmit antenna and the $j$th receive antenna is denoted by $h_{ij,iV}$ for the vertical component and $h_{ij,iH}$ for the horizontal component. The cross-components are denoted by $h_{ij,iH}$ and $h_{ij,iV}$, respectively.

A. Channel XPD

An electromagnetic wave can due to reflections change its polarization state as it propagates through a channel. A common way of describing a channel’s ability to separate V and H polarizations is the so called channel XPD factor which is for the fast fading part of the channel defined as

$$
\text{XPD} = \frac{\text{E}\{|\tilde{h}_{V,V}|^2\}}{\text{E}\{|\tilde{h}_{H,H}|^2\}} = \frac{\text{E}\{|\tilde{h}_{H,H}|^2\}}{\text{E}\{|\tilde{h}_{V,H}|^2\}},
$$

(2.3)

where a symmetric leakage is assumed. The same “symmetry” assumption was made for V/H polarized waves in [3] and it is also motivated by the measurements reported in [1] where the leakage from V to H and H to V have the same power on average. We also choose to define the variable $\alpha$, $0 < \alpha \leq 1$, which corresponds to the part of the radiated power that is coupled from V to H and vice versa. There is perfect discrimination between the V and H polarized components as $\alpha \to 0$, and there is “leakage” between the polarizations when $0 < \alpha \leq 1$. The relation between the channel XPD and $\alpha$ is, thus, given by

$$
\text{XPD} = \frac{1 - \alpha}{\alpha}, \quad 0 < \alpha \leq 1
$$

(2.4)

where we have used the following normalizations

$$
\text{E}\{|\tilde{h}_{V,V}|^2\} = \text{E}\{|\tilde{h}_{H,H}|^2\} = 1 - \alpha,
$$

(2.5)

$$
\text{E}\{|\tilde{h}_{H,H}|^2\} = \text{E}\{|\tilde{h}_{V,H}|^2\} = \alpha.
$$

(2.6)

The above normalizations are motivated by power or energy conservation arguments. That is, the channel can not introduce more energy to the transmitted signal and with this normalization the power is conserved by subtracting from the co-polarized component the corresponding amount of power $\alpha$ that has leaked into the cross-polarized component. This normalization is of great importance when comparing DP to SP systems. For example, one can draw totally different conclusions from using other normalizations. In, e.g., [7], the average power $\alpha$ that is coupled from one polarization to the other is not compensated for in the co-polarized channel, i.e., $\text{E}\{|\tilde{h}_{V,V}|^2\} = \text{E}\{|\tilde{h}_{H,H}|^2\} = 1$, independently of $\alpha$. Thus, the introduction of $\alpha > 0$ makes the channel introduce more energy which of course is good for the communication performance but unfortunately contradicts the physics behind wave propagation.

Similarly, we define the channel XPD for the fixed part of the channel as

$$
\text{XPD}_{f} = \frac{1 - \alpha f}{\alpha f},
$$

(2.7)

where we have used the following normalizations

$$
|\tilde{h}_{V,V}|^2 = |\tilde{h}_{H,H}|^2 = 1 - \alpha f,
$$

(2.8)

$$
|\tilde{h}_{H,H}|^2 = |\tilde{h}_{V,V}|^2 = \alpha f.
$$

(2.9)

B. Channel correlation

For example, when the channel is not rich enough, i.e., when there is not enough scattering to decorrelate the elements of the channel matrix and/or when the antenna spacing is too small, the elements of the SS-SP MIMO channel matrix will be correlated. We define the transmit, $t_s$, and receive, $r_s$, spatial and co-polarized correlation coefficients as

$$
t_s = \frac{\text{E}\{\tilde{h}_{iV,V}\tilde{h}_{iV,V}\}}{\sqrt{1 - \alpha} \cdot \sqrt{1 - \alpha}}, \quad i \neq j
$$

(2.10)

$$
r_s = \frac{\text{E}\{\tilde{h}_{iH,H}\tilde{h}_{iH,H}\}}{\sqrt{1 - \alpha} \cdot \sqrt{1 - \alpha}}, \quad i \neq j.
$$

(2.11)

Similarly, we define the transmit, $t_p$, and receive, $r_p$, polarization correlation coefficients as

$$
t_p = \frac{\text{E}\{\tilde{h}_{iV,V}\tilde{h}_{iH,H}\}}{\sqrt{\alpha(1 - \alpha)} \cdot \sqrt{\alpha(1 - \alpha)}},
$$

(2.12)

$$
r_p = \frac{\text{E}\{\tilde{h}_{iV,V}\tilde{h}_{iH,H}\}}{\sqrt{\alpha(1 - \alpha)} \cdot \sqrt{\alpha(1 - \alpha)}}.
$$

(2.13)

For example, the measurements reported in [5] showed that the average envelope correlations (worst case) were all less

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than 0.2, and, in fact, all of the reported measurements in [1] showed that \( t_p \approx r_p \approx 0 \).

By using the symmetry that is inherited from the above XPD and correlation assumptions we can express the DP \( 2 \times 2 \) MIMO channel as

\[
\tilde{H}_{tx} = \Sigma \circ \left( C_{r_p}^{1/2} W_{2 \times 2} C_{t_p}^{1/2} \right), \quad i = 1, 2
\]  

(2.14)

where

\[
\Sigma = \begin{bmatrix} \sqrt{1-\alpha} & \sqrt{\alpha} \\ \sqrt{\alpha} & \sqrt{1-\alpha} \end{bmatrix},
\]

\[
C_{r_p} = \begin{bmatrix} r_p & 0 \\ 0 & r_p \end{bmatrix}, \quad C_{t_p} = \begin{bmatrix} t_p & 0 \\ 0 & t_p \end{bmatrix},
\]

are the polarization leakage, receive polarization correlation and the transmit polarization correlation matrices, respectively, and \( W_{2 \times 2} \) is a \( 2 \times 2 \) matrix with i.i.d. and circularly symmetric complex Gaussian noise entries with zero mean and variance \( 1/2 \) in each real dimension. Furthermore, the symmetry assumption also allows us to express the \( 4 \times 4 \) MIMO channel matrix in the following compact way

\[
\tilde{H} = (1_{2 \times 2} \otimes \Sigma) \circ \left( C_{r_p}^{1/2} W_{4 \times 4} C_{t_p}^{1/2} \right),
\]  

(2.15)

where \( C_r = C_{r_r} \otimes C_{r_p} \) and \( C_t = C_{t_r} \otimes C_{t_p} \) are the receive correlation and transmit correlation matrices of the \( 4 \times 4 \) MIMO channel with two spatially separated dual polarized antennas on each side. The matrix \( 1_{2 \times 2} \) is a \( 2 \times 2 \) matrix consisting of ones and the spatial correlation matrices are, respectively, given by

\[
C_{r_r} = \begin{bmatrix} 1 & r_r \\ r_r & 1 \end{bmatrix}, \quad C_{t_r} = \begin{bmatrix} 1 & t_r \\ t_r & 1 \end{bmatrix}.
\]  

(2.16)

However, in this study we only work with \( 2 \times 2 \) systems by choosing the appropriate elements from the larger \( 4 \times 4 \) channel matrix.

### III. ANTENNA MODELING

#### A. Polarization

The DP antenna is simply modeled by an antenna polarization matrix and the polarization matrix contains two unit norm column vectors that represent the polarization vectors. That is, the transmit antenna matrix is given by

\[
\bar{A}_{tx} = [p_1, p_2],
\]  

(3.1)

where \( p_i, \quad i = 1, 2 \) are the unit norm polarization vectors. The receive antenna matrix also contains two polarization vectors and it is denoted by \( \bar{A}_{rx} \). Two polarizations are said to be orthogonal if their inner-product is zero, i.e., \( p_1 \) is orthogonal to \( p_2 \) if \( p_1^H p_2 = 0 \). However, one can also consider polarization parallelity which is a measure of non-orthogonality, and in this work we consider DP antennas that do not necessarily have orthogonal polarizations. In fact, the parallelity, \( p \), is a model parameter and it is here defined as

\[
p = \frac{|p_1^H p_2|}{||p_1|| ||p_2||} = |p_1^H p_2|,
\]  

(3.2)

and with this definition we have that \( 0 \leq p \leq 1 \).

#### B. Coupling

A DP antenna with a nonzero parallelity will experience coupling between the polarizations. Since the two polarizations of a DP antenna may have a common phase center they can not offer array gain. It can be shown using matched network theory that this coupling will lead to a complex scaling of the original polarization vectors. Due to the limited space this derivation is however omitted, and we simply state that the coupling between the polarizations scales the polarization matrix in the following way

\[
\bar{A}_{tx} = \frac{1}{\sqrt{1 + p}} A_{tx} = \frac{1}{\sqrt{1 + p}} [p_1, p_2],
\]  

(3.3)

where the arbitrary complex phase rotations have been removed since the channel capacity is invariant to unitary transformations. Note that the solution is intuitively appealing since for zero parallelity there is no coupling thanks to perfect orthogonality and therefore no power loss (the scaling factor equals unity), and for full parallelity the transmitted signal power is distributed evenly (the scaling factor equals \( 1/2 \)) over the two identical polarizations. The same solution and argumentation also holds for the receive polarization matrix \( \bar{A}_{rx} \).

### IV. THE OVERALL CHANNEL INCLUDING ANTENNAS

The signal model is given by

\[
x = \sqrt{\frac{\sigma^2}{n_{tx}}} H_{tot} s + n,
\]  

(4.1)

where \( x \) is the received signal vector, \( H_{tot} \) is the total channel matrix (now including the transmit and receive polarization matrices), \( s \) is the transmitted signal vector and normalized as \( \mathbb{E} [ s^H s ] = n_{tx} \) where \( n_{tx} \) is the number of transmit antenna ports, \( n \) is a noise vector with i.i.d. and circularly symmetric complex Gaussian entries with variance \( 1/2 \) for each real dimension, \( \sigma^2 \) is the average transmitted signal power. The total \( 2 \times 2 \) MIMO channel matrix is given by

\[
H_{tx,tot} = \bar{A}_{tx}^H H_{tx} H_{tot} \bar{A}_{tx}, \quad i = 1, 2.
\]  

(4.2)

That is, the transmitted signal vector \( s = [s_1, s_2]^T \) is first multiplied by \( \bar{A}_{tx} \) which in the \( 2 \times 2 \) MIMO case is a linear combination of the two polarization vectors. This signal is then multiplied by the random channel matrix \( H_{tx} \) and the resulting and now randomly polarized signal is finally probed with the receive antenna matrix by multiplying by \( \bar{A}_{tx}^H \) which yields the signals on the two receive antenna ports.

The \( 2 \times 2 \) MIMO channel is easily extended to a \( 4 \times 4 \) channel by introducing an additional spatially separated DP antenna on each side and the extended antenna matrix is then given by

\[
\bar{A}_{4 \times 4,tx} = \begin{bmatrix} \bar{A}_{tx} & 0_{2 \times 2} \\ 0_{2 \times 2} & \bar{A}_{tx} \end{bmatrix},
\]  

(4.3)
where the same extension also holds for the receive antenna matrix. Thus the overall \(4 \times 4\) MIMO channel is given by

\[
H_{tot} = \bar{A}^H H_{tx} A_{rx} \bar{H} A_{rx}^{H 4 \times 4} = \begin{bmatrix}
[\bar{A}^H H_{tx} A_{rx} \bar{H} A_{rx}^{H 4 \times 4}] & \bar{A}^H H_{tx} A_{rx} \bar{H} A_{rx}^{H 4 \times 4}
\end{bmatrix},
\]

where \(H\) is the \(4 \times 4\) channel given by (2.2) which has the covariance structure given by (2.15).

If we denote the channel under study by \(H_c\) (constructed by choosing the appropriate elements from \(H_{tot}\)), then the signal model is given by replacing \(H_{tot}\) by \(H_c\) in (4.1). The channel \(H_c\) is always assumed to be perfectly known to the receiver and we study the channel capacity for two cases of channel state information (CSI): (i) the channel is perfectly known to the transmitter (CSIT); (ii) the channel is unknown to the transmitter (no CSIT).

### V. Results

In this section we evaluate the capacity for different cases such as varying correlations, XPD, parallelity, and polarizations, etc. In all Ricean fading examples the fixed \(2 \times 2\) channel components belong to

\[
H_c \in \left\{ \left[ \begin{array}{c}
\sqrt{1 - \alpha_f} & \sqrt{\alpha_f} \\
\sqrt{\alpha_f} & \sqrt{1 - \alpha_f}
\end{array} \right], \left[ \begin{array}{c}
\sqrt{1 - \alpha_f} & \sqrt{1 - \alpha_f} \\
\sqrt{1 - \alpha_f} & \sqrt{1 - \alpha_f}
\end{array} \right] \right\}
\]

where

\[
\text{XPD}_f = \frac{1 - \alpha_f}{\alpha_f} = 15 \text{ dB}.
\]

The DP channel has the left fixed component and the SS-SP channel has the one to the right. Note that the fixed part of the DP channel is full rank while the fixed part of the SS-SP channel is rank deficient, which is likely in LOS scenarios.

1) Capacity vs. XPD: In Fig. 5.1 we see the capacity (with and without transmitter CSI) for a Rayleigh fading DP channel with V and H polarized antennas, and for a SS-SP channel with two V polarizations on each side. The parameter settings are given in the figure caption. Low XPD implies strong leakage between the polarizations and high XPD implies weak leakage. Both systems benefit from high XPD, whereof especially the SS-SP system shows good performance since its array gain provides a 3 dB advantage over the DP system. However for low XPD, the SS-SP setup shows its weakness which is that it can yield very low capacity due to polarization mismatch. In fact, it can yield zero capacity if the XPD tends to negative infinity (total mismatch due to full polarization leakage).

Also note that the SS-SP system’s array gain advantage is lost at high SNR which is due to only logarithmic capacity increase.

2) Capacity vs. K-factor: The capacity is here evaluated with respect to channel K-factor and SNR for the Ricean fading channel. In Fig. 5.2 we see that the SS-SP system once again thanks to its array gain outperforms the DP system at low SNR. However for high SNR and dominating LOS components (which usually appear simultaneously), the DP system yields best performance since it provides full rank channels while the uninformed transmitter performs suboptimal transmit signaling over the rank deficient SS-SP LOS channel.

3) Different polarizations and varying parallelity: In this example we study the DP \(2 \times 2\) MIMO channel and we let the transmitter have orthogonal polarizations while the receiver has varying parallelity. The transmitter has three different DP setups: (i) V and H; (ii) clockwise and counterclockwise circular; and (iii) \(\pm 45^\circ\). We let the receiver have one polarization which is fixed and matched to the transmitter, i.e., \(H\) in case (i), circular in case (ii), and \(-45^\circ\) in case (iii). Then we choose the second polarization such that it fulfills the chosen parallelity. However, for case (i) it is linear, case (ii) elliptical, and for case (iii) also linear. The channel is a Ricean fading channel. The remaining parameter settings are given in the caption of Fig. 5.3. We see from the figure that the choice of polarizations influences the capacity for high parallelities and that high K-factors together with low parallelity is good since it provides a fixed channel with two equally strong subchannels. Note how an increase in parallelity deteriorates the capacity by reducing the overall channel rank and by introducing power loss due to coupling between the non-orthogonal polarizations.

4) Receivers with random polarizations and parallelities: In a down link scenario the base station might have well designed DP antennas, i.e., the polarizations are well defined and likely to be orthogonal to each other (at least in the directions of the strongest lobes). However, user devices might not have well defined polarizations and the polarizations typically vary. Also the parallelity varies for different directions and devices. We model the random polarizations and parallelities on the receiver side such that for each channel realization a random polarization and a random parallelity are generated. The random polarizations are drawn from a cosine distribution (in more detail, the ellipse angle follows a cosine distribution and the major axis angle follows a uniform distribution) and the parallelities are drawn from a uniform distribution. Fig. 5.4 shows the capacity with respect to XPD for various \(2 \times 2\) MIMO systems. For the DP setup, the transmitter has perfect V/H polarizations whereas the receiver has a DP antenna with two random polarizations and a random parallelity. In the SS-DP setup the transmitter has a spatial separation between the V and H polarizations and the receiver has also a spatial separation between the two random polarizations with random parallelity (the only difference compared to the first setup is the spatial separation between the polarizations, i.e., less correlation and no coupling). The SS-SP setup is a transmitter with two spatially separated V polarizations and a receiver with two spatially separated but identical random polarizations, which implies a higher risk for polarization mismatch in the receiver. For each use of the channel, new independent polarizations and parallelities are drawn. As one can see, the best performance is achieved by the SS-DP setup. This is due to its robustness to polarization mismatch, less correlation, and zero coupling. It is also interesting to see that every setup except for the SS-SP one benefit from high XPD. In Fig. 5.1, which was generated by having perfect orthogonal polarizations in both transmitter and receiver, all setups benefit from high XPD. The explanation to this is that the higher the XPD is the less
Polarization leakage there is. Thus, a transmitted V polarized wave will also be received as a V polarized wave. Now, if the receiver only has the ability to sense the channel by using SP antennas, there is higher likelihood of polarization mismatch and it is therefore better for a SP receiver to sense a channel with lower XPD since the conveyed wave will then also have a H polarized component.

VI. CONCLUSIONS

The combined SS-DP MIMO channel was modeled and the model included parameters such as channel correlation, XPD, polarization states, polarization parallelity, and coupling. From this model, the capacity for DP and SS-SP systems was evaluated and compared. It was found that SS-SP systems during idealized conditions will outperform DP systems thanks to its array gain. Array gain, however, is only beneficial at low SNR and it has little contribution to the capacity at high SNR. At high SNR it is more important to have parallel channels which DP MIMO provides. Furthermore, SS-SP systems are sensitive to polarization mismatch (due to polarization leakage in the channel and/or rotations of a mobile) and they often exhibit high channel correlation due to insufficient antenna spacings combined with small angular spreads, which both will deteriorate the capacity. It is in these more realistic scenarios that a DP setup is beneficial since it offers diversity/multiplexing over orthogonal polarizations, which especially is beneficial in LOS conditions with high SNR. Finally, it was found that polarization parallelity degrades the capacity and that the choice of polarization states affects the performance for non-zero parallelities.

REFERENCES