Performance of Closely Spaced Multiple Antennas for Terminal Applications

Anders Derneryd, Jonas Fridén, Patrik Persson, Anders Stjernman

Ericsson AB, Ericsson Research
SE-417 56 Göteborg, Sweden
{anders.derneryd, jonas.friden, patrik.c.persson, anders.stjernman}@ericsson.com

Abstract — The influence of correlation, efficiency, and antenna branch signal unbalance on receive diversity gain and Shannon capacity is investigated. A case study using closely spaced dipoles, with mutual coupling included, is used for the analysis. These results are compared with results derived from measured embedded radiation patterns of dual-antenna mock-ups.

I. INTRODUCTION

The antenna system is becoming a key component in mobile communications systems when new applications demand higher data rates, and as the coverage requirement increases. This poses significant challenges for antenna system designers of both the access points and the user terminals. Multiple antennas offer additional degrees of design freedom that are used to increase the capacity, extend the coverage, or reduce the interference. The advantage of a Multiple Input Multiple Output (MIMO) system is to exploit the spatial channel for increased spectral efficiency [1], [2], [3], [4].

A fundamental characteristic for increased capacity in MIMO systems is the independence between the different data streams. One source of correlation between the streams is antenna mutual coupling in compact devices. As a result, compact multi-antenna designs must make a trade-off between size and performance. Antenna mutual coupling changes the input impedance and distorts the radiation pattern when the antenna elements are placed in close proximity.

This paper presents performance results of closely spaced multiple antennas for terminal applications. As a case study parallel dipole antennas at different separation are used. Finally, the results are compared with measurements on multi-antenna terminal mock-ups.

II. CLOSELY SPACED DIPOLES

In order to capture the effects of a physical multi-antenna realization while keeping the possibility of easy parameterization, arrays of dipoles including matching network and mutual coupling are analyzed [5]. Two parallel dipoles are evaluated in terms of antenna branch signal correlation, efficiency, diversity gain, and Shannon capacity in the frequency range from 1.05 GHz to 2.55 GHz. The dipole length is fixed at 78 mm, which corresponds to 0.47 wavelengths at 1800 MHz. The dipole separation is varied from 2 mm to 120 mm. The radiation patterns are calculated including coupling effects and combined with a spectrum of incident waves in order to evaluate the performance in a uniform 3D propagation environment.

III. CORRELATION AND EFFICIENCY

The correlation between the branch signals received at the antenna ports is one parameter that is of interest when estimating diversity gain. Signal correlation due to coupling between the antenna elements are calculated from the realized embedded gain radiation patterns and the power spectrum of the incident field [6], [7]. An advantage with this method is that the influence of the propagation environment on the correlation is included. The antenna radiation efficiency and the loading of the antenna ports are implicitly included.

The antenna branch signals are calculated by combining the embedded far field radiation pattern with the complex incident rays. The magnitude of the complex correlation is shown in Fig. 1, when the dipoles are located in a uniform 3D power spectrum of the incident field. For each separation, the dipoles are matched for no return loss at broadside radiation and a fixed frequency (1800 MHz) corresponding to equal magnitude and phase excitation. At the dipole spacing 25 mm (0.15 wavelengths at 1800 MHz), the magnitude of the complex correlation is 0.53 (marked by a circle in the figure) in a uniform 3D environment with equal antenna branch powers.

![Fig. 1] Magnitude of the complex correlation for two parallel dipoles in a uniform 3D environment as a function separation. Frequency in MHz is used as a parameter.
The antenna radiation efficiency, Fig. 2, and the loading of the antenna ports are implicitly included in the correlation calculation. The radiation efficiency for matched antennas reduces with a more compact antenna design as the coupling losses increases. The radiation efficiency at 25 mm dipole spacing and 1800 MHz is -1.8 dB corresponding to about 65% as marked by a circle in the figure.

Fig. 2 Efficiency of two parallel dipoles as a function of separation. Frequency in MHz is used as a parameter.

IV. RECEIVE DIVERSITY

A. Balanced Receive Branch Power

A CDF-plot (cumulative distribution function) of the received signal branch powers for two parallel dipoles located in a uniform 3D environment is shown in Fig. 3. The simulation is based on 50 000 realizations with 20 incoming rays at each channel sample point. The dipoles at 1800 MHz are separated 25 mm (0.15 wavelengths). The two left solid lines correspond to the two dipole branches (|v_1|^2 and |v_2|^2, respectively) while the solid right curve is the total combined received signal using MRC (|v_1|^2 + |v_2|^2) (maximum ratio combining) [8, p. 558]. As seen, the separate antenna branch powers are equal in the analyzed environment. The diversity gain, measured as the increase in total received signal power compared to the strongest branch signal, at the 1% level is 10.7 dB assuming MRC.

The diversity gain at the 1% level for all dual dipole cases analyzed is presented in Fig. 4 as a function of the magnitude of the complex correlation. Three different combining methods are displayed; selection combining (max (|v_1|^2, |v_2|^2) / |v_1|^2), equal gain combining (\(\sqrt{2} (|v_1|^2 + |v_2|^2) / |v_1|^2\)), and maximum ratio combining (\((|v_1|^2 + |v_2|^2) / |v_1|^2\)). A complex correlation of 0.53 gives a diversity gain of 10.7 dB (marked by a circle in the figure) using MRC and balanced received branch powers, which agrees with the results given in Fig. 3. Selection combining diversity gain is about 2 dB less and equal gain combing falls between the MRC and the selection combiner performance.

B. Unbalanced Receive Branch Power

The eigenvalues of the covariance matrix of the antenna branch signals is of fundamental importance for multiple antenna system capacity. Any lossless signal combination, a unitary matrix multiplication, will leave these eigenvalues unchanged. Hence, the received signal power of the maximum ratio combining and the Shannon capacity are invariant with respect to unitary combinations of the antenna branch signals.

Since the covariance matrix is Hermitean, four real parameters are enough for parameterization of a two stream system. By using the average signal power, \(P_{av}\), in antenna branches 1 and 2, the power unbalance, \(\chi\), between signal in antenna branches, and the complex signal correlation, \(\rho_{12}\), the covariance matrix can be written as [8, p. 568]

\[
\text{Cov} = \frac{2P_{av}}{1 + \chi} \left[ \begin{array}{cc} 1 & \rho_{12} \sqrt{\chi} \\ \rho_{12} \sqrt{\chi} & \chi \end{array} \right]
\]

Fig. 3 CDF of received antenna branch signal powers in a uniform 3D environment at 1800 MHz for two parallel dipoles separated 25 mm.

Fig. 4 Diversity gain at the 1% level for two parallel dipoles in a uniform 3D environment as a function of the antenna branch signal complex correlation.

The eigenvalues of the covariance matrix of the antenna branch signals is of fundamental importance for multiple antenna system capacity. Any lossless signal combination, a unitary matrix multiplication, will leave these eigenvalues unchanged. Hence, the received signal power of the maximum ratio combining and the Shannon capacity are invariant with respect to unitary combinations of the antenna branch signals.
with eigenvalues (let $|\rho_{1,2}| = \rho$)

$$\lambda_{1,2} = P_{av} \left(1 \pm \sqrt{1 - \frac{4\chi(1-\rho^2)}{(1+\chi)^2}}\right) \quad (2)$$

Note that any lossless (unitary) transformation of the antenna branch signals will leave the eigenvalues unchanged. Therefore, any function of the eigenvalues will remain constant, e.g., condition number, average power, signal power level after maximum ratio combining, and Shannon capacity. Specifically, the geometric mean of the eigenvalues, $\nu$, becomes

$$\nu = \sqrt{\lambda_1 \lambda_2} = P_{av} \frac{2\sqrt{\chi(1-\rho^2)}}{1+\chi} \quad (3)$$

An alternative invariant is the condition number, which is the ratio of the eigenvalues, $R = \lambda_1 / \lambda_2 \geq 1$. From (2) it follows that

$$\frac{(1+\chi)^2}{\chi} = \frac{(1+R)^2}{R}(1-\rho^2) \quad (4)$$

Two extreme results of unitary transformations are identified from (4).

- Totally uncorrelated receive branch signals with maximum power unbalance, i.e., correlation $\rho = 0$ and the power ratio $\chi = \chi_0 = R$.
- Perfectly balanced receive branch signals with maximum correlation, i.e., the power ratio $\chi = 1$ and correlation $\rho = \rho_0 = (R-1) / (R+1)$.

Note that in the case of equal signal power ($\chi = 1$) in both antenna branches, (2) reduces to

$$\lambda_{1,2} = P_{av} (1 \pm \rho) \quad (5)$$

A balanced antenna system ($\chi = 1$) with correlation $\rho_0$ can be traded against an uncorrelated system ($\rho = 0$) with branch power unbalance $\chi_0 = (1+\rho_0) / (1-\rho_0)$ [8, p. 569]. A correlation of 0.53 ($\rho_0$) in an environment with balanced receive branch power can be traded against a difference of 5.1 dB ($\chi_0$) between two antenna branch signals with zero correlation. A way to generate uncorrelated receive branch signals is to form sum and difference signals. The corresponding orthogonal radiation patterns at 1800 MHz with the dipoles terminated in matched loads are plotted in Fig. 5.

As above, the diversity gain can be calculated as the increased total received signal power compared to the strongest branch signal. A CDF plot is shown in Fig. 6 of the received sum and difference signal powers at 1800 MHz when the dipoles are separated 25 mm (0.15 wavelengths). The two left solid lines correspond to the sum signal ($\frac{1}{2} |v_1 + v_2|^2$) and the difference signal ($\frac{1}{2} |v_1 - v_2|^2$), respectively, while the solid right curve is the total signal using MRC ($\frac{1}{2} |v_1|^2 + |v_2|^2$). As seen, the generated branch powers are unbalanced with a difference of 5.1 dB ($= 10 \log [(1+\rho_0) / (1-\rho_0)]$). The diversity gain at the 1% level is 8.9 dB assuming MRC. The estimated diversity performance is usually calculated in relation to the performance of the strongest branch, a 100% efficient reference antenna in free space, or an ideal Rayleigh curve [8, p. 556], [9], [10].

The diversity gain at the 1% level for all analyzed dipole cases is presented in Fig. 7 as a function of the unbalance between the completely uncorrelated received signals. Three different combining methods are analyzed as before, selection combining ($\frac{1}{2} \max (|v_1 + v_2|^2, |v_1 - v_2|^2) / \max (\frac{1}{2} |v_1 + v_2|^2, \frac{1}{2}|v_1 - v_2|^2)$), equal gain combining ($\frac{1}{4} (|v_1 + v_2| + |v_1 - v_2|)^2 / \max (\frac{1}{2} |v_1 + v_2|^2, \frac{1}{2}|v_1 - v_2|^2)$), and maximum ratio combining ($\frac{1}{2} |v_1 + v_2|^2 + \frac{1}{2} |v_1 - v_2|^2) / \max (\frac{1}{2} |v_1 + v_2|^2, \frac{1}{2}|v_1 - v_2|^2)$). The dotted line is the theoretical curve for a Rayleigh distributed signal.

A branch signal unbalance of 5.1 dB gives a diversity gain of 8.9 dB using MRC (marked by a circle in the figure), which agrees with the results presented in Fig 6. Selection combining diversity gain is about 2 dB less and equal combing falls between the MRC and the selection combiner performance as seen before.
A summary plot of the diversity gain assuming uncorrelated and correlated antenna branch signals are shown in Fig. 8. The total received power is the same in both cases although the diversity gain is different depending on the reference level used.

C. Mock-ups

A more relevant performance measure is the effective diversity gain that compares the combined received power to the received power of a 100% efficient antenna [9]. The effective diversity gain at the 1% level is plotted in Fig. 9 as a function of the geometric mean of the eigenvalues assuming a uniform 3D environment and MRC at the receiver. The performance of measured mock-ups such as phones, PDAs, and notebooks is also included and it is seen that the performance closely follows the theoretical curve as given by the dipole cases. On the contrary, plotting the effective diversity gain as a function of mean received power and condition number is presented in Fig. 10. The radius of the circular discs is proportional to the effective diversity gain. As expected, devices with high losses due to mismatch, coupling, and ohmic losses do achieve low effective diversity gains. Moreover, it seems like the variation in effective diversity gain is minimized along curves of constant $\nu$ (geometric mean of eigenvalues (3)), suggesting that the effective diversity gain depends more or less only on this parameter.

V. SHANNON CAPACITY

Another parameter of importance is the Shannon capacity. The average Shannon capacity is plotted in Fig. 11 as a function of SNR (Signal to Noise Ratio) for four transmission cases, namely SISO, SIMO with MRC at the receiver end, 2 x 2 MIMO with un-informed transmitter, and 2 x 2 MIMO with water filling at the transmitter [3]. The propagation environment is modelled as two uncorrelated uniform 3D Rayleigh channels with 10 independent rays per channel and 10 000 independent realizations. The frequency is 1800 MHz and the antenna separation is 25 mm (0.15 wavelengths).
Receive diversity (SIMO) seems to give a constant improved capacity across a large range of SNR values. As seen, MIMO transmission in noise-limited environments is mainly advantageous at SNR levels above 10 to 15 dB. The 2 x 2 MIMO mode almost doubles the Shannon capacity compared to the SISO case as expected. The average 2 x 2 MIMO capacity of two closely spaced parallel dipoles (solid lines) is 85% of the theoretical maximum capacity of two crossed lossless dipoles (dashed lines). The 2.5 dB difference in SNR to achieve the maximum theoretical MIMO capacity is due to correlation, and mismatch, coupling, and ohmic losses.

The maximum theoretical complex correlation in a lossy environment is (solve for $l_1 = 1$ and $P_{sw} = \eta$ in (5)) [11]

$$\rho_{\text{max}} = \frac{1 - \eta}{\eta}$$ (6)

This limitation is included as a solid line far to the right in the figure.

A number of dual-antenna mock-ups have been evaluated in a simulated uniform 3D environment. Measured radiation patterns are combined with the channel model. The 2 x 2 Shannon capacity of these devices is also plotted in Fig. 12. The performance of the mock-ups falls close to the low correlation part of the diagram. The dipole case discussed previously is marked by a larger circle in the figure.

VI. CONCLUSIONS

Simulation results show that, in practical cases, both effective diversity gain and 2 x 2 MIMO Shannon capacity in a noise-limited environment are more or less uniquely determined by the geometric mean of the eigenvalues of the antenna branch signal covariance matrix.

For a balanced dual-antenna system with a given correlation, another system with the same capacity and no correlation can be found at the cost of introducing branch signal unbalance. At low SNR values, antenna systems with different correlations and branch signal unbalance but the same mean efficiency provide similar capacity.

REFERENCES